# Homework 4:
# Velocity Kinematics and Jacobians

MEAM 520, University of Pennsylvania
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This assignment is due on **Friday, November 2 (updated)**, by 5:00 p.m. sharp. You should aim to turn the paper part in during class the day before. If you don't finish until later in the day, you can turn it in to Professor Kuchenbecker's office, Towne 224. Late submissions will be accepted until 5:00 p.m. on Monday, November 5, but they will be penalized by 25%. After that deadline, no further assignments may be submitted.

You may talk with other students about this assignment, ask the teaching team questions, use a calculator and other tools, and consult outside sources such as the Internet. To help you actually learn the material, what you write down should be your own work, not copied from a peer or a solution manual.

## Written Problems *(60 points)*

This entire assignment is written and consists of two significantly adapted problems from the textbook, *Robot Modeling and Control* by Spong, Hutchinson, and Vidyasagar (SHV). Please follow the extra clarifications and instructions on both questions. Write in pencil, show your work clearly, $\boxed{\text{box your answers}}$, and staple your pages together.

1. **Adapted SHV 4-20, page 160 – Three-link Cylindrical Manipulator** ***(30 points)***
   The book works out the DH parameters and the transformation matrix $T_3^0$ for this robot on pages 85 and 86; you are welcome to use these results directly without rederiving them.

   (a) Use the position of the end-effector in the base frame to calculate the $3 \times 3$ linear velocity Jacobian $J_v$ for the three-link cylindrical manipulator of Figure 3.7 on page 85.

   (b) Use the positions of the origins $o_i$ and the orientations of the z-axes $z_i$ to calculate the $3 \times 3$ linear velocity Jacobian $J_v$ for the same robot. You should get the same answer as before.

   (c) Find the $3 \times 3$ angular velocity Jacobian $J_\omega$ for the same robot.

   (d) Find this robot's $6 \times 3$ Jacobian $J$.

   (e) Imagine this robot is at $\theta_1 = \pi/2$ rad, $d_2 = 0.2$ m, and $d_3 = 0.3$ m, and its joint velocities are $\dot{\theta}_1 = 0.1$ rad/s, $\dot{d}_2 = 0.25$ m/s, and $d_3 = -0.05$ m/s. What is $v_3^0$, the linear velocity vector of the end-effector with respect to the base frame, expressed in the base frame? Make sure to provide units with your answer.

   (f) For the same situation, what is $\omega_3^0$, the angular velocity vector of the end-effector with respect to the base frame, expressed in the base frame? Make sure to provide units with your answer.

   (g) Use your answers from above to derive the singular configurations of the arm, if any. Here we are concerned with the linear velocity of the end-effector, not its angular velocity. Be persistent with the calculations; they should reduce to something nice.

   (h) Sketch the cylindrical manipulator in each singular configuration that you found, and explain what effect the singularity has on the robot's motion in that configuration.

2. **Adapted SHV 4-18, page 160 – Three-link Spherical Manipulator *(30 points)***
The book does not seem to work out the forward kinematics for this robot anywhere. Please use the diagram on the left side of Figure 1.12 on page 15 in SHV to define the positive joint directions and the zero configuration for the robot. If we additionally choose the $x_0$ axis to point in the direction the robot arm points in the zero configuration, you can calculate that the tip of the spherical manipulator is at $[x \;\; y \;\; z]^T = [c_1c_2d_3 \;\; s_1c_2d_3 \;\; d_1 - s_2d_3]^T$. In this expression $\theta_1$, $\theta_2$, and $d_3$ are the joint variables; $s_i$ is $\sin\theta_i$ and $c_i$ is $\cos\theta_i$; and $d_1$ is a constant.

   (a) Calculate the $3 \times 3$ linear velocity Jacobian $J_v$ for the spherical manipulator with no offsets shown in the left side of Figure 1.12 on page 15 of SHV. You may use any method you choose.

   (b) Find the $3 \times 3$ angular velocity Jacobian $J_\omega$ for the same robot.

   (c) Find this robot's $6 \times 3$ Jacobian $J$.

   (d) Imagine this robot is at $\theta_1 = \pi/4$ rad, $\theta_2 = 0$ rad, and $d_3 = 1$ m. What is $\omega_3^0$, the angular velocity vector of the end-effector with respect to the base frame, expressed in the base frame, as a function of the joint velocities $\dot{\theta}_1$, $\dot{\theta}_2$, and $\dot{d}_3$? Make sure to provide units for any coefficients in these equations, if needed.

   (e) For the same configuration described in the previous question, what is $v_3^0$, the linear velocity vector of the end-effector with respect to the base frame, expressed in the base frame, as a function of the joint velocities $\dot{\theta}_1$, $\dot{\theta}_2$, and $\dot{d}_3$? Provide units for any coefficients in these equations, if needed.

   (f) What instantaneous joint velocities should I choose if the robot is in the configuration described in the previous questions and I want its tip to move at $v_3^0 = [0 \text{ m/s} \;\; 0.5 \text{ m/s} \;\; 0.1 \text{ m/s}]^T$? Make sure to provide units with your answer.

   (g) Use your answers from above to derive the singular configurations of the arm, if any. Here we are concerned with the linear velocity of the end-effector, not its angular velocity. Be persistent with the calculations; they should reduce to something nice.

   (h) Sketch the cylindrical manipulator in each singular configuration that you found, and explain what effect the singularity has on the robot's motion in that configuration.

   (i) Would the singular configuration sketches you just drew be any different if we had chosen different positive directions for the joint coordinates? What if we had selected a different zero configuration for this robot? Explain.

3. **Optional Extra Credit – Visualizing the Linear Velocity Jacobian *(unknown points)***

   If you have time and interest, feel free to try this optional extra-credit problem. Modify your solution for the PUMA robot animation in Homework 3 (`puma_robot_yourpennkey.m`) in the following ways:

   - Rename the file `jacobian_yourpennkey.m`
   - Eliminate the spherical wrist, so that end-effector is at the origin of frame 3 (the wrist center).
   - Remove the offsets by setting $b$ and $d$ to zero. This should give you an articulated manipulator.
   - Change the zero configuration as follows: when all three angles are zero, the arm should be horizontal and pointing in the direction of the positive $x_0$ axis. Although this is not what is shown in Figure 4.5 on page 145 in SHV, I think this is the zero configuration they used.
   - Use the expression for $J_{11}$ on page 144 in SHV to augment the visualization of the robot with three lines that go through the tip of the robot and show the direction in which the tip will move if you have only one non-zero joint velocity. Make the line for $\dot{\theta}_1$ red, the line for $\dot{\theta}_2$ green, and the line for $\dot{\theta}_3$ blue. Feel free to adjust other plotting parameters as needed.
   - Check your solution with the provided motion modes, and feel free to create a new motion mode that showcases the Jacobian augmentation you added.

   Submit your code as an attachment to an email to `meam520@seas.upenn.edu` with the subject *Jacobian Extra Credit: Your Name*, replacing *Your Name* with your name.